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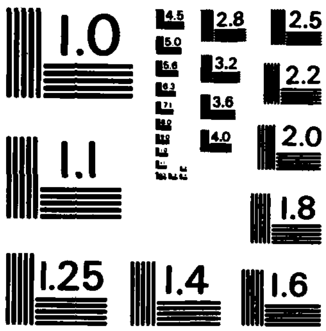

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Major engineering projects have focused attention on problems created by the freezing of water in earth materials. These problems received detailed attention during the design of the Alaska Natural Gas Transportation System. They also have received attention in the design and construction of pipelines in Siberia and Canada. The problems arise from the tendecy of ice to form in lenses segregated from the surrounding soil. As these lenses grow, they are capable of generating large pressures that disturb the

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surface of the ground. The term frost heaving is used to describe this phenomenon. Efforts to design roadways, airfields, buildings, powerlines, and other structures to prevent damage from frost heaving date from the early 1900s.

Because of the seriousness of the problem, much time and money have been expended in searching for methods of predicting and controlling ice segregation and frost heaving. Unfortunately, misunderstanding and controversy have developed due to inadequate knowledge of the physical processes involved, and due to the widely differing backgrounds of scientists and engineers who have been owrking on the problem. Recognizing this, the Committee on Permafrost of the Polar Research Board established an Ad Hoc Study Group on Ice Secgregation and Frost Heaving to summarize the current state of knowledge and to identify areas of controversy or uncertainty that additional research might resolve. This report is the result. Its intent is to provide a clearer understanding of the general nature of the field, of the controversial questions that remain, and of the research required to resolve them.

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ICE SEGREGATION AND FROST HEAVING

FINAL REPORT

AD HOC STUDY GROUP ON ICE SEGREGATION AND FROST HEAVING

1984

U.S. ARMY RESEARCH OFFICE

CONTRACT DAAG29-81-K-0163

NATIONAL RESEARCH COUNCIL

ICE SEGREGATION AND FROST HEAVING

Ad Hoc Study Group on Ice Segregation and Frost Heaving
Committee on Permafrost
Polar Research Board
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National Research Council

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ICE SEGREGATION AND FROST HEAVING

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FOREWORD

Major engineering projects have focused attention on problems created by the freezing of water in earth materials. These problems received detailed attention during the design of the Trans-Alaska oil pipeline, and additional aspects have been encountered during the design of the Alaska Natural Gas Transportation System. They also have received attention in the design and construction of pipelines in Siberia and Canada. The problems arise from the tendency of ice to form in lenses segregated from the surrounding soil. As these lenses grow, they are capable of generating large pressures that disturb the surface of the ground. The term frost heaving is used to describe this phenomenon. Efforts to design roadways, airfields, buildings, powerlines, and other structures to prevent damage from frost heaving date from the early 1900s.

Because of the seriousness of the problem, much time and money have been expended in searching for methods of predicting and controlling ice segregation and frost heaving. Unfortunately, misunderstanding and controversy have developed due to inadequate knowledge of the physical processes involved, and due to the widely differing backgrounds of scientists and engineers who have been working on the problem. Recognizing this, the Committee on Permafrost of the Polar Research Board established an Ad Hoc Study Group on Ice Segregation and Frost Heaving to summarize the current state of knowledge and to identify areas of controversy or uncertainty that additional research might resolve. This report is the result. Its intent is to provide a clearer understanding of the general nature of the field, of the controversial questions that remain, and of the research required to resolve them.

It is the hope of the Ad Hoc Study Group and the Polar Research Board that this assessment will aid in the resolution of the outstanding problems and thus assist the scientific and engineering community in preventing and controlling the adverse consequences of frost effects.

Charles R. Bentley, Chairman
Polar Research Board

CONTENTS

EXECUTIVE SUMMARY

Frost heaving caused by ice segregation occurs throughout the cold regions of the world where sustained low temperatures are commonly encountered. Annual ground displacements of several inches with cyclic differential ground pressures of hundreds of pounds per square inch are common. Adverse effects are widespread and costly. Attempts to avoid the damaging effects of frost heaving often dominate design considerations and, thus, the cost of highways, airfields, and the foundations of major structures. Large sums of money have been spent during the past two decades on the development of procedures and techniques designed to understand and predict the complicated processes of ice segregation and frost heaving. In addition to federal and state agencies, many engineering firms and consortia of firms in private industry are contributing to this effort.

During the past five years alone, it is estimated that in excess of $50 million has been spent for the design, construction, and instrumentation of experimental field sites at remote locations and for data collection and data analysis to obtain information on ice segregation and frost heaving needed for the design of the Alaskan Natural Gas Transportation System. At least an equivalent amount of money has been expended on collecting geotechnical data along proposed right of ways and in evaluating technical approaches to the prediction of frost heaving hazards for pipeline design.

This report and the general recommendations that follow are intended to provide guidance and assistance to federal and state agencies, private firms, and consortia in whose interest it is to continue research in the scientific, geotechnical, and other engineering aspects of ice segregation and frost heaving. The intent is to accelerate progress in attaining a complete understanding of the phenomena associated with seasonal and long-term ice segregation and frost heaving and the application of this knowledge in the various planning, design, and maintenance activities associated with buildings, facilities, airfields, roads, utilities, pipelines, and other structures wherever the need exists.

Engineering hazards due to the displacements and pressures generated by ice segregation and frost heaving, together with the adverse effects of accumulations of segregated ice in freezing soil, are common throughout most of North America, Europe, and Asia. Mountainous areas that are subject to seasonal freezing and thawing also are affected. Recognition of damage caused by frost heaving dates back to Roman times. Increasingly

it has been realized that, although the engineering techniques required to avoid or protect against frost heaving damage may be expensive, failure to protect adequately against these effects often leads to much higher costs for subsequent repair or reconstruction.

Systematic scientific investigations of frost heaving began in the early 1900s. Major advances in the understanding of ice segregation and frost heaving were made during the 1950s. During the 1970s, large engineering projects in North America, Europe, and Asia produced an intensification of effort. As a result, the principal processes of the ice segregation that cause frost heaving have been recognized and are now being defined in detail. Freezing air temperatures create a thermal gradient that induces upward heat flow. Soil water freezes near the ground surface, and segregated ice crystals form and grow. They then coalesce into lamelar lenses (ice segregation). Ice lens growth is a consequence of a continuous upward flow of water from below. Heat flow and the flow of soil moisture occur in the same direction and are interdependent. During this process a balance is set up between the dissipation of the latent heat of freezing and the upward flow of soil water. The upward displacement of the soil surface (frost heaving) is a consequence of the growth of a series of ice lenses. When displacement is restricted, "heaving pressures" develop; under certain conditions, very large frost heaving pressures are possible.

The process of ice segregation involves complex interrelation-ships between the ice, an unfrozen liquid phase, and the bulk pore water. Complex latent heats of phase change as well as variable interfacial energies between phases are involved. Although there is general agreement on the existence of the various phases and the general concepts for which governing equations and "rules of behavior" are being sought, we still are far from the detailed understanding needed to explain fully ice segregation and to predict adequately rates, total and differential displacements, and pressures that result from frost heaving.

In order to organize the conceptual information available and to utilize it for the prediction of the various consequences of ice segregation and frost heaving, the techniques of mathematical simulation and numerical modeling have been applied. There is widespread agreement that the models of ice segregation and frost heaving must include appropriate equations describing heat flow and the flow of soil water and that they must deal with total and differential heave displacement and with frost heaving pressures. Beyond this general agreement, however, there are strongly divergent views on many fundamental questions; for example, the form and basic structure of "an adequate model" is being debated. There is also disagreement on how to deal with surcharge and overburden pressures during the ice segregation process, etc. Finally, elaboration of existing one- and two-dimensional models to three dimensions is an undertaking of great complexity and difficulty. There are many detailed questions to be answered before significant progress toward a complete mathematical simulation of ice segregation and frost heaving can be developed.

1. We recommend a continued concentrated research effort to understand the basic processes of ice segregation and frost heaving. Among the most critical unresolved questions and problems are:

- What are the basic relationships that govern the coupled transport processes (liquid water, heat, water vapor, ice) in ice segregation?
- How are overburden pressures related to ice segregation and frost heaving?
- Do the same basic relationships between overburden pressure and other parameters that characterize frost heaving apply under both static and dynamic conditions?
- What are the important basic principles and relationships governing long-term, as distinct from seasonal, frost heaving?
- What are the maximum attainable frost heaving pressures under long-term frost heaving conditions and how are they best controlled?
- What are the sources of the energy involved in frost heaving and how are they partitioned among the coupled processes involved?
- Solute redistribution occurs during ice segregation. More information is needed on the basic principles and proccesses.
- How is regelation involved in the buildup and maintenance of frost heaving pressures?
- What determines the temperatures and governs the phenomena of ice nucleation in freezing soils?

2. We recommend a continued intensive research effort to develop predictive numerical models that, by simulating ice segregation and frost heaving, can provide a means of consolidating all the processes and interactions involved. This will continue to aid in attaining a better and more complete understanding of frost heaving and for assessing its impact in various settings and circumstances. Among the most critical unresolved questions and problems pertaining to this aspect are:

- What are the essential and most appropriate parameters required?
- What are the essential criteria for ice segregation and the formation of ice lenses or enlarging ice masses?
- How should the coupled transport processes (heat, liquid water, water vapor, ice) be most effectively and appropriately formulated?
- How can the locations of enlarging ice lenses be most effectively specified and the processes of "secondary heaving" be better defined?
- Must the partition of energy among the various processes involved be specifically recognized and dealt with? If so, how?
- Are there useful characteristic frequency distributions and autocorrelative functions that can be generally agreed upon and worked into the various approaches to numerical simulations of frost heaving?
- In both seasonal and long-term simulations, what is the best technique for dealing with and scaling time?

Progress in solving all of these difficult and controversial questions is crucial to the attainment of a complete, rational theory of ice segregation and frost heaving. Such a theory, however, is needed for the improved mathematical simulations and numerical models capable of producing reliable assessments of frost heaving hazards in a variety of locations and circumstances where engineering design and construction must take place.

3. A common terminology, clearly relating to and bridging existing terminologies in the various disciplines, is needed to address the phenomena of ice segregation and frost heaving and to facilitate a more efficient and rational discussion of the processes, consequences, and methods of ameliorating their adverse effects. Therefore, we recommend that the several national scientific and professional societies involved give explicit attention to this need and encourage efforts to achieve an appropriate, common terminology.

4. Improved instrumentation to measure the critical phenomena associated with ice segregation and frost heaving is needed. Most of this instrumentation must be developed for unique uses and circumstances. We recommend that this need be explicitly recognized by funding bodies and that the design and fabrication of needed instrumentation, in addition to the procurement of more conventional and readily available instrumentation, be a prominent part of the budgets of ice segregation and frost heaving research projects.

5. We recommend that field and laboratory studies of ice segregation and frost heaving be better coordinated in order to accelerate progress in the verification and improvement of the various mathematical models currently available and to accelerate the quantification of the functions and parameters of the various interrelated processes. We recommend that this coordination be a part of the protocol of all major research projects dealing with problems of ice segregation and frost heaving.

In recent years professional societies have organized several conferences to facilitate the exchange of knowledge and data on the problems of ice segregation and frost heaving. However, much more could be done at relatively little cost during the planning stages of major research projects. The needed coordination could be accomplished without compromising the essential requirements of obtaining and managing proprietary data. Indeed, if attention is given to this recommendation, the value of the proprietary information obtained could be vastly increased.

INTRODUCTION

A large fraction of the land surface north of the equator is unfrozen in summer but in winter freezes to some depth, however slight. Farther north the ground is normally frozen all the time but thaws to some depth in summer. When moist or wet soil freezes, it often is accompanied by expansion due to a process known as frost heaving, while thawing of frozen soil often is accompanied by a process known as thaw-subsidence. Heaving is always associated with the appearance of bodies of more or less pure segregated ice within or outside the soil while thaw-subsidence is associated with the melting of segregated ice. Each of these processes has unwanted or damaging effects on engineering works and on agricultural crops and enterprises.

The appearance of segregated ice, however, may not be accompanied by heave if there happens to be a corresponding amount of internal consolidation of the soil proper. Processes involved in the formation of segregated ice and the associated frost heave are much more complicated than generally is supposed. Consequently, the effects are difficult to predict with the levels of confidence that an engineer requires. While a number of theories have been proposed to explain ice segregation and frost heave, there is little agreement among scientists with respect to which, if any, of these theories is physically consistent and sound; there is uncertainty as to how to settle that disagreement and the degree to which all the difficult questions must be settled in order to develop reliable means of predicting heave that will best serve the most critical needs of geotechnical engineers.

Ice segregation and frost heaving occur naturally wherever the climate is cold enough to freeze moist, fine-grained soils. Nearly everyone living in the northern and southern temperate zones has experienced the effects of ice segregation and frost heaving through the destruction of roads and highways, the displacement of foundations, the jamming of doors, the misalignment of gates, and the cracking of masonry. Many people often have simply and mistakenly assumed that these effects result solely from the expansion of pore water on freezing. When confined, water can rupture pipes, break bottles, and crack rocks as it freezes. However, most of the destructive effects of frost heaving are caused by "ice segregation," a complex process that results from the peculiar behavior of water and other liquids as they freeze within porous materials. In particular, water is drawn to the freezing site from

elsewhere by the freezing process itself. When this water accumulates as ice, it forces the soil apart, producing expansion of the external soil boundaries, as well as internal consolidation. The dynamic process of ice segregation and the expansion resulting from freezing of the in situ pore water, together, cause frost heaving.

The costs of frost damage in cold climates have not been fully determined. Although these costs are large, estimates have not been tabulated to date, partly because of the difficulty of recognizing all the instances of frost effects and a tendency to ascribe many of the consequences of frost heaving to other causes. For example, the breakup of roads in spring is often attributed to a combination of snow melt and impeded drainage. In fact, the real cause is the impregnation of road bases with segregated ice during winter. During the spring thaw, the ice melts, weakening the base and causing the pavement to fail when loaded by traffic. Even though during the last 20 years or more, design procedures for highways, buildings, powerlines, and other structures have been dramatically improved, frost heaving and thaw weakening are still common occurrences. Much yet remains to be done to improve procedures to mitigate frost action and reduce the high costs of foundation construction in cold climates. Although sufficient understanding now exists to avoid damage due to ice segregation and frost heaving in most cases, the design procedures required are often conservative and, therefore, costly to employ. Also, substantial damage to plants and crops occurs each year wherever frost heaving can occur. Further refinements in our knowledge of ice segregation and frost heaving are needed to make design criteria more discriminate and, thus, less expensive and to mitigate adverse effects on crops and other agricultural enterprises.

Most of the cold regions of the world where the effects of ground freezing are critical to design considerations lie in the northern hemisphere continents of North America, Europe, U.S.S.R., China, and Asia. The geographical areas subject to annual frost heaving and frost effects are shown in Figure 1. In these areas, the design of structures requires careful attention to avoid damage due to frost action. The needs of highway engineers have been served by rules of thumb (based on grain-size distributions) suggested 50 years ago by Casagrande. The rules are imperfect and surely result in excessively costly construction much of the time, but since there can be no assurance that adequate precautions have been taken the rest of the time, they are followed wherever possible. Casagrande's rules were devised for use in projects that are close analogues of highway construction. Special problems of frost heaving induced by buried chilled pipelines, for example, cannot be solved using highway design criteria, yet such projects are being planned for Alaska and Canada with estimated construction costs in the range of tens of billions of dollars. Clearly a much better understanding is required for large projects of this type.

In the 1950s, defense construction led to improvements in geotechnical engineering techniques developed during World War II to cope

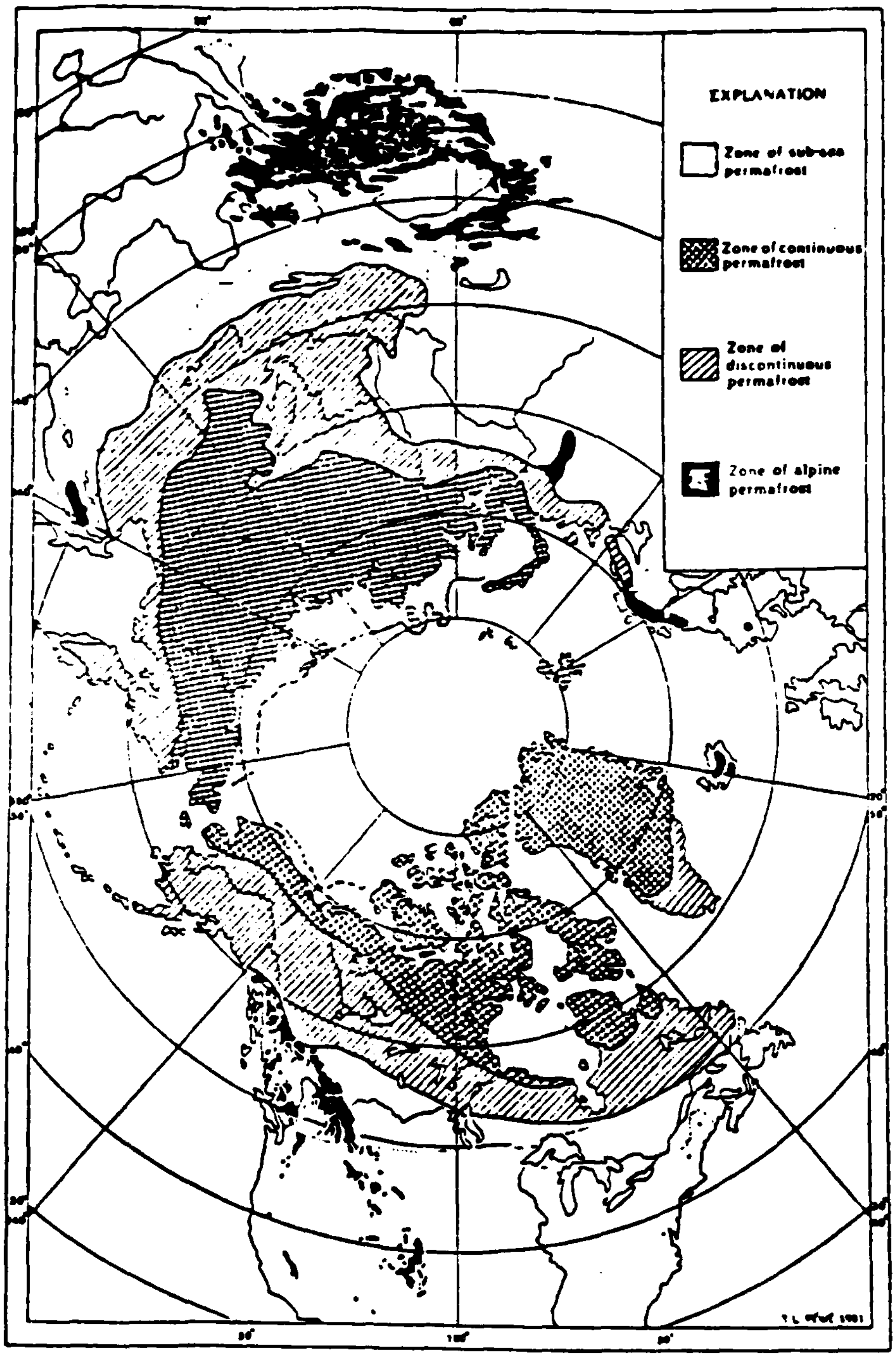

Figure 1. Distribution of Permafrost in the Northern Hemisphere (Compiled by T. L. Pewe 1983).

with the adverse effects of freezing and thawing of the ground. During this period, there was a great demand for improved standards for road and runway construction on soils subject to freezing and susceptible to frost heaving. These developments and the requirements for building foundations on permafrost stimulated research into the rates and extent of freezing and thawing, the dependence of these rates on climate and ground conditions, and the fundamental principles governing the freezing and thawing of porous media. The understanding gained during this period, although useful, was largely qualitative; the engineering solutions devised as a result were relatively simple, but they have been effective in many diverse circumstances.

In the 1970s a change of emphasis occurred. During this period, natural resource development in regions of permafrost led to renewed attention to frost heaving phenomena. The largest of the projects of this period, the Trans-Alaska oil pipeline, not only required detailed techniques to avoid disturbing the permafrost, but careful attention to problems associated with frost heaving in the soil region experiencing annual summer thaw and winter refreezing and throughout extensive areas of discontinuous permafrost. This attention continues in connection with the design of cold, buried natural gas pipelines, buried tanks for the storage of liquefied natural gas, and the artificial freezing of soil for purposes of stabilization. Interest is not confined to North America. In northern areas of the Soviet Union, a number of large-diameter pipelines have been constructed in the last six years, and multiple, large-diameter pipes running in close proximity pass from the Urengoy fields toward the south. The multiple parallel lines provide a redundancy that is partly related to anticipated difficulties associated with the possible thawing of permafrost and to freeze-thaw phenomena.

Pipelines are particularly susceptible to damage in freezing or thawing ground because of the extensive area and variety of soil materials and hydrologic conditions along these routes. Successful construction and reliable operation require appropriate geotechnical designs that deal adequately with freeze-thaw phenomena. In many projects, the final costs of pipelines are increased significantly because of the need to deal with the problems associated with the freezing and thawing of the active layer in permafrost and of seasonally frozen ground in areas where permafrost is absent.

In the case of natural gas pipelines in permafrost regions, plans may involve chilling the gas to below freezing temperatures to increase capacity and to prevent thawing of permafrost. In regions of discontinuous permafrost, however, chilling results in potential danger of frost heaving wherever the pipe passes through unfrozen ground. This problem is further compounded by the very long-term freezing period, usually many years. In the case of the proposed Alaska Natural Gas Transportation System, as much as 20 percent of the alignment in Alaska is subject to this hazard.

Problems associated with frost heaving were not satisfactorily resolved in the design of a proposed Mackenzie Valley, large-diameter gas pipeline before this project was cancelled. During public review, it became apparent that the possible build-up of frost heave pressures due to ice segregation was being underestimated. The uplifting forces that could be developed from the freezing of soil around the pipe were initially considered to be small enough to be counteracted by the weight of overlying materials. Additional investigations, however, demonstrated the possible development of very large pressures. As a consequence, major design changes including more than surcharge effects appeared to be necessary. This matter still is controversial.

The design of the Alaska Natural Gas Transportation System (known as the Alaska Highway Gas Pipeline in Canada) has been under development in the United States and Canada for some time. During the last five years, a program involving full-scale test sites in Alaska has been in progress to develop appropriate designs and design procedures for the regions of continuous and discontinuous permafrost. Expenditures on these test sites probably have exceeded $50 million. At the same time, additional analyses involving mechanical properties of frozen ground, as well as heat flow and moisture migration, have all been conducted at considerable expense.

Another major activity requiring improved knowledge of the ice segregation process is the practice of artificial ground freezing. This is an industry that has been expanding in recent years as economic factors have made it competitive with other soil stabilization techniques. Artificial ground freezing is used to stabilize soft soils and prevent seepage during excavation for structures, tunnels, and shafts. Calculation of the thickness of freezewalls requires accurate methods of estimating moisture and heat flow during freezing. However, precautions must be taken to prevent frost heave and thaw settlement which could cause extensive damage to adjacent structures.

As the behavior of natural soils during freezing is still largely unknown, the need for continuing fundamental investigations remains. A basic requirement is the capacity to make quantitative predictions of the magnitude of ice segregation (including the extent to which this continues in already-frozen ground) and of the pressures that can be generated at specific locations and in specific circumstances. These predictions require an ability to predict accurately the flow of both heat and moisture as the ground freezes and thaws in a wide variety of geomorphic and climatic conditions.

CURRENT KNOWLEDGE

Early Investigations

Frost heaving was first subjected to detailed study by Taber (1916, 1929, 1930). He observed that when soil materials are frozen from the top down, under conditions allowing water to move upward within the pore space from below, the soil surface moved upward and segregated ice layers appeared. As freezing progressed, he could observe no marked change in the rate of upward movement as one ice lens ceased to grow and another one appeared beneath it. He also noted similar behavior could be observed in soil materials wetted with liquids such as benzine or nitrobenzine. This finding is important because these liquids differ from water in that they contract in changing from the liquid to the solid phase. This observation establishes that ice segregation and the resultant frost heaving are due principally to the imbibition and local accumulation of the pore fluid and not simply to the expansion of the pore fluid during solidification.

The principal processes of ice segregation as they now are understood are illustrated in Figure 2. Air temperatures below freezing create freezing at the soil boundary and induce a thermal gradient in the soil that causes water flow toward the soil boundary. Freezing is initiated and the ice crystals grow to form lamelar masses. Ice lens growth is achieved by the upward movement of water toward the freezing surface. A balance is struck between the rate of heat release due to ice lens growth and the dissipation by other processes of the latent heat of freezing. The displacement of soil toward the soil surface is a consequence of ice lens enlargement and, when restricted, leads to the development of heaving pressures. Soil displacement occurs after soil consolidation is achieved and continues as long as a favorable balance exists between the rate of heat removal and the upward movement of soil water. Confining pressures and soil characteristics also are important. When the heat and moisture fluxes become sufficiently unequal, new ice lenses form at sites below, where a favorable balance between these two interacting fluxes is possible.

The ground surface depicted in Figure 2 shows a vertical cut to emphasize that (a) flow lines for heat conduction usually exit perpendicular to the soil surface, (b) flow lines for soil water movement in general are parallel to the flow lines for heat conduction, and (c) the resulting planar ice lenses tend to be perpendicular to the soil water

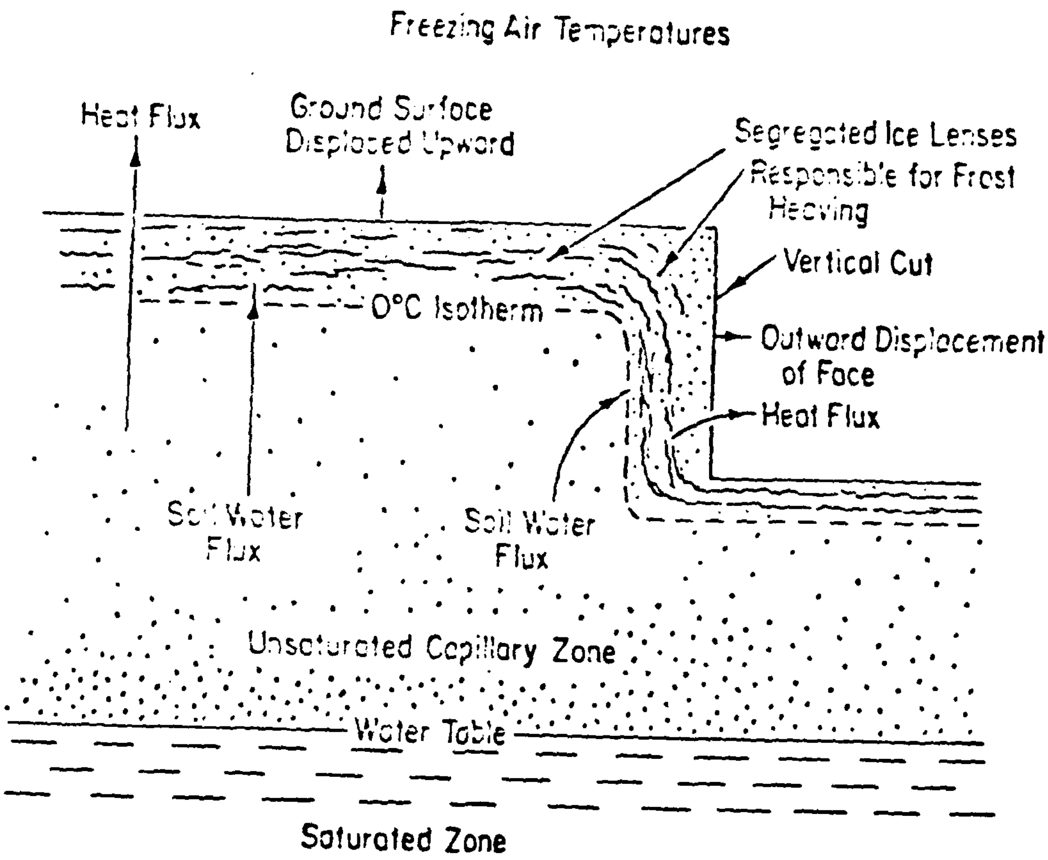

Figure 2. A schematic diagram of ice segregation and frost heaving (Anderson et al. 1984).

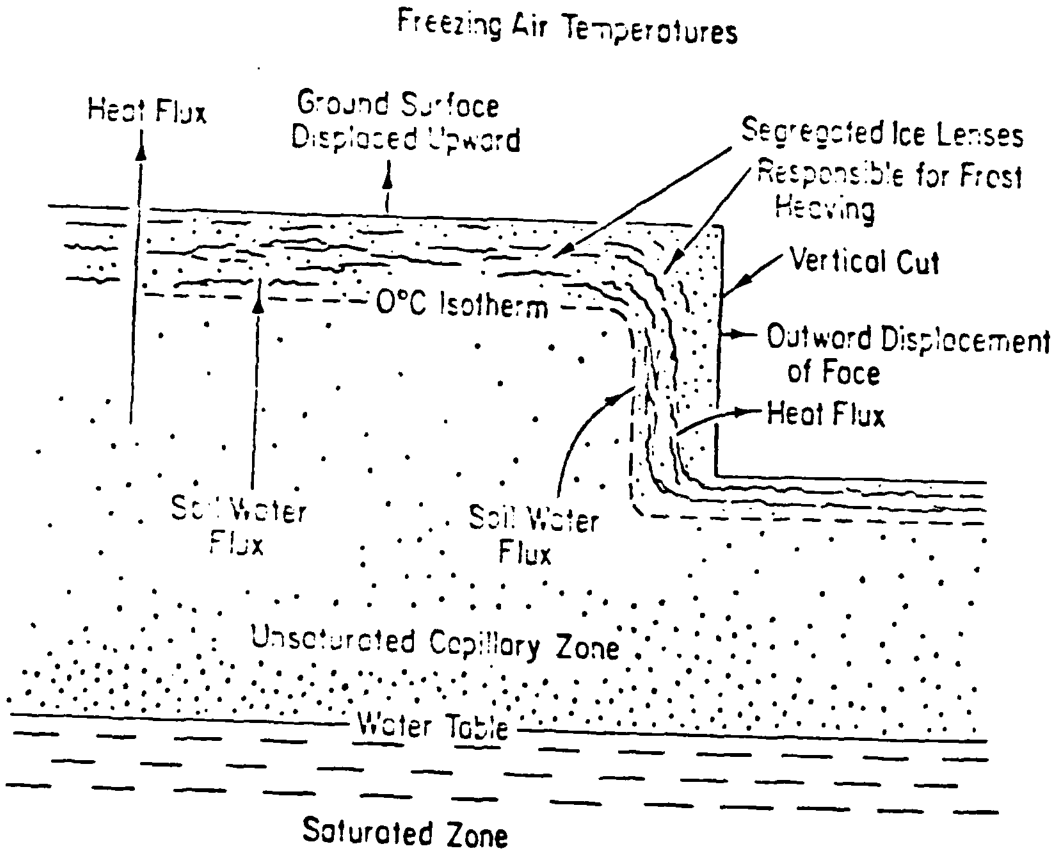

Figure 2. A schematic diagram of ice segregation and frost heaving (Anderson et al. 1984).

flux and thermal flux flow lines. Thus, it is possible for the process of ice segregation to produce both vertical and horizontal earth displacements. The general rule is: displacements due to ice segregation tend to be perpendicular to the ground surface. In general, the nearer the water table is to the surface, the more readily water is transported through the partially saturated, capillary zone to the freezing zone.

In addition to obtaining an early general comprehension of these basic processes and relationships, Taber also recognized that the ice lenses, for the most part, are separated from the solid particles making up the soil structure during their enlargement. This is the only way he could explain the clear, segregated ice lenses that he observed to form. He reasoned that as heat was conducted toward the surface there must be an unfrozen film of water supporting the ice lenses as they formed, separating them from the soil fabric. This film, he visualized, maintained itself by imbibing additional water from the moist, unsaturated soil below, setting up a hydraulic gradient that induced the continual flow of water required to perpetuate the process. Beskow (1935) enlarged upon this basic concept and proposed that the "capillary" characteristics of soils, determined primarily by grain-size distribution, govern the extent to which soils may heave when subjected to freezing conditions.

Some peculiarities of the process of frost heaving can be illustrated by a description of the behavior of "coarse," "medium," and "fine-grained" soils in either "open" or "closed" systems that may be either "saturated" (i.e., virtually air-free) or "unsaturated," and for cases in which the load supported by soil in the zone of ice formation is "small" or "large." Coarse sand or gravel, a silt, and a swelling clay are examples of what is meant by coarse, medium, and fine, respectively. Small loads may approach zero; large loads may be of the order of 1 MPa. A closed system is one that is isolated from an external source of water, something that is not true of an open system.

If loads are large enough, and if the systems are saturated and closed, all soils behave superficially in ways that, from the engineer's viewpoint, are practically the same. At any stage of freezing, the heave can be equated to the volume change of that part of the initial water present that has changed to ice. In this case, there may be marked differences in the appearances of the coarse, medium, and fine soils, with a lot more ice segregation in the very fine (highly compressible) soil than in the two virtually incompressible soils which, accordingly, contain much more "pore ice."

If the systems are open but otherwise as above, then the coarse soil will not heave at all; neither will the medium soil, but the fine one may behave essentially as it did when closed, assuming that time allowed is relatively small. If the loads are very small, and the systems are again closed and saturated, the coarse soil will heave almost exactly as if the load was very large, and so will the very fine soil. The medium soil will heave quite a lot more than either of the other two, however. On the other hand, if the loads are small and the systems are open and saturated

or near saturation, the coarse soil still will not heave at all; the clay will behave much as it always does, but the medium soil can produce truly spectacular heaving!

Energies at the Interface and Unfrozen Water

The earliest observations and insights on ice segregation have been confirmed, expanded, and elaborated upon by others. For example, the "frost susceptibility" of soils was extensively investigated by Casagrande and later by Kaplar (1968); the role of surface tensions, particularly the surface tension of the ice-water interface, was examined by Gold (1957), Miller et al. (1960), Everett (1961), Koopman's and Miller (1966), Miller (1973, 1977, 1978), and Penner (1977), among others. The nature, properties, and behavior of the transition zone at the interface separating the ice lenses from the soil mass also have been the subjects of many subsequent investigations. For example, Bouyoucos and McCool (1916) and Bouyoucos (1917) established that frozen soils retained a fraction of their total water content in the unfrozen state and indirectly correlated the unfrozen water with the interaction occurring at the interface between the mineral matrix and the soil water. Similar investigations were conducted by Beskow (1935) and Tsytovich (1945).

The early calorimetric method of determining the quantity of unfrozen water in frozen soil was refined by Martynov (1956), Nersesova and Tsytovich (1966), and Williams (1963, 1964ab, 1967). Additional methods of verifying the existence of the unfrozen water in capillary spaces and interfacial films and of defining the properties of the unfrozen liquid water in these domains were devised by Anderson and Hoekstra (1965ab), Anderson (1966, 1967a), and Anderson and Tice (1972). Less direct methods that corroborate the existence of the unfrozen water have been contributed by Hoekstra (1965), Hoekstra and Chamberlain (1964), Nakano et al. (1972), and Patterson and Smith (1980).

As a result of these investigations, the liquid-like characteristics of the water in frozen soil materials have been described and well established. Electrical conductance measurements have demonstrated the mobility of ionic species in frozen soil (Hoekstra 1965, 1969a; Murrmann et al. 1970). The fact that water in frozen soil can be mobilized by electrical or thermal gradients demonstrates the continuity and the high molecular mobility of the water in the interface between the ice and the soil particles. That soil particles can migrate through ice masses proves that individual mineral particles are free of any but the most feeble and transitory connections to the ice phase (Hoekstra 1969a). Nuclear magnetic resonance (NMR) data indicate that the transition zone retains its liquid-like characteristics to very low temperatures (-40°C or lower). The NMR data permit an estimate of the viscosity of the unfrozen

water at the interface. Although somewhat unrealistic because of the small physical dimensions involved, it has been suggested that the viscosity of the unfrozen water is about 600 times greater than that of ordinary water, comparable to the viscosity of glycerol (Fripiat et al. 1965; Anderson and Morgenstern 1973). Measured diffusion coefficients in frozen soils are consistent with this view (Murrmann et al. 1970). This conclusion is not fully established, however.

Typical relationships between the quantity of unfrozen water in frozen soil and temperature are shown for three representative soil materials in Figure 3 (Tice et al. 1978). The figure shows that the effect of varying total water and ice contents is minor. The quantity of unfrozen water present depends principally on the temperature. When the data of Figure 3 are normalized by dividing the unfrozen water contents at any temperature by the specific surface area of the soil materials, the data tend to coalesce. Calculated thicknesses of the unfrozen water films resulting from this treatment range from approximately two diameters of the water molecule at temperatures of -10°C and lower to progressively larger dimensions at temperatures approaching the melting point. Other factors, such as salinity and pressure also act to influence the thickness of these unfrozen water films and the quantity of unfrozen water present at any given temperature. An increase in salinity or pressure increases the quantity of unfrozen water at a given temperature (Banin and Anderson 1974; Anderson 1967b).

It is possible to visualize a variety of physical arrangements of the interconnected, unfrozen water films throughout a frozen soil mass. The experimental investigation and analysis of Koopmans and Miller (1966) relate both temperature and pressure to unfrozen water content in saturated soils. This work suggests that under many common conditions near the site of active segregation a substantial amount of pore water remains unfrozen in the smaller pores, in addition to that in adsorbed films. Simulations based on the concepts of Miller (1978) support this view (O'Neill and Miller, 1982, 1984). This means that unfrozen pore space, as well as the unfrozen films, may contribute significantly to the pathways that liquid water may take in feeding ice lens growth.

Hydrological Aspects

The role that ice lenses play in restricting infiltration during the winter and spring melt is important in governing surface water runoff and groundwater recharge. Water from the snowpack can either return to the atmosphere, infiltrate into subsurface systems, or leave as surface runoff. The infiltration rate into seasonally frozen soils is controlled by the fraction of the pore space that is occupied by ice and by the barrier to flow that is created by ice lenses. Seasonally frozen soils

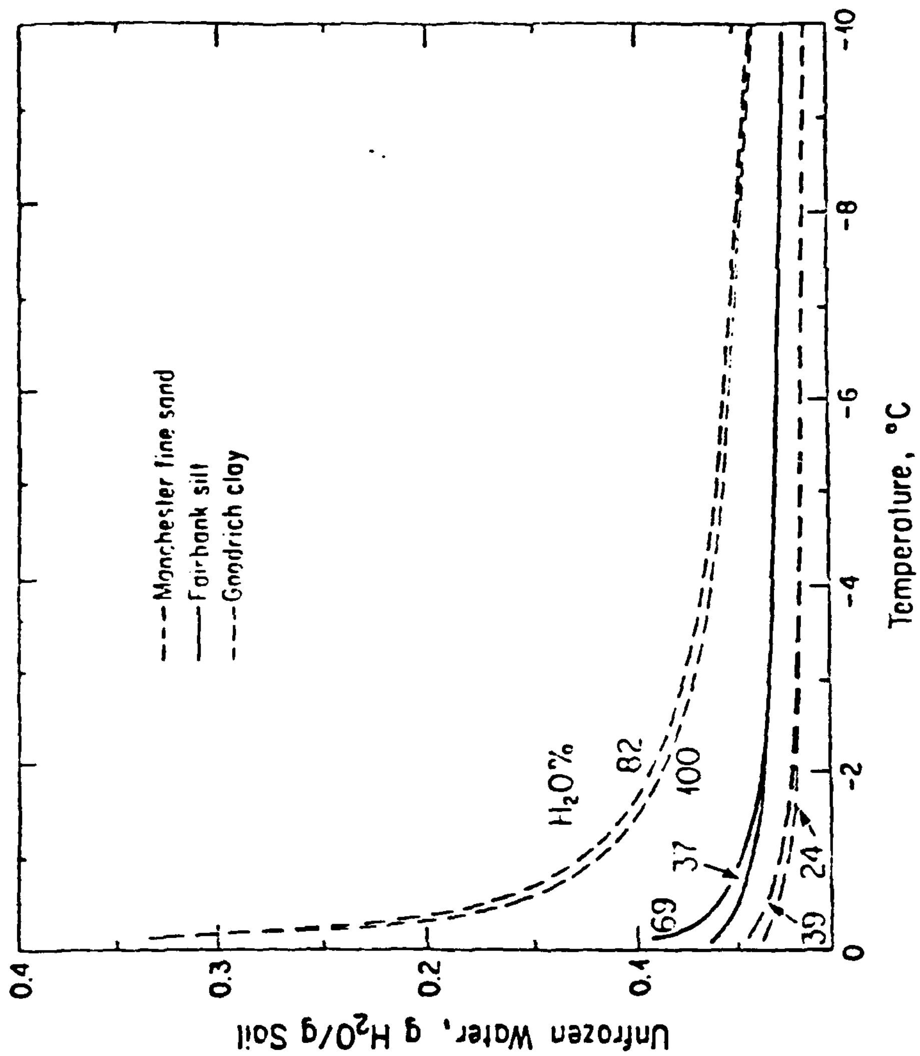

Figure 3. A plot of unfrozen water content versus temperature for several representative materials (Andersland and Anderson 1978).

with low moisture contents have relatively high infiltration rates because there is little ice blocking the soil pores. This facilitates groundwater recharge. Soils containing large quantities of ice are less permeable; in this case surface water runoff is enhanced.

It is a generally accepted view that ice lens development occurs somewhat behind the freezing front. As can be visualized from Figure 2, when unfrozen water is drawn upward to the freezing front in response to a hydraulic gradient, there may be a distinct zone between the freezing front and the region where ice lenses develop; this is referred to as the "frozen fringe." The hydraulic conductivity of the frozen fringe is one of the principal governing factors that determines where ice lens growth actually occurs. In addition to a dependence on the hydraulic conductivity of the frozen fringe, Konrad and Morgenstern (1981) have related the position where ice lens development occurs to the temperature gradient. Penner (1982) has shown that the position of the ice front is also related to the overburden pressure; increasing the overburden pressure lowers the temperature at the ice lens.

The hydraulic gradient is composed principally of a gravitational component and a "soil water potential" component. The gravitational potential gradient is constant, is independent of the freezing process, and is always directed downward. The soil water potential gradient, on the other hand, depends on the freezing process and may be orders of magnitude greater than the gravitational component. Other potential energy gradients that may be involved, such as that arising from osmotic forces, are important when solutes are present. All things considered, however, net movement of water within the soil generally is from warmer to colder areas. Thus, the direction of movement of soil moisture normally coincides with the direction of heat transfer.

The process of lens initiation and growth within the frozen soil behind a frozen fringe has been referred to, or designated as, "secondary heave" (Miller 1972). The possibility of continual redistribution of water among the ice lenses also is an important facet of frost heaving phenomenology. Water can migrate from ice crystals and ice lenses within the frozen zone from frozen, but warmer parts of the soil, to colder areas. This redistribution of soil water within the frozen zone must proceed at a much slower rate than the upward movement of water to the freezing front, however, because of the much lower hydraulic conductivities of the frozen material. The hydraulic conductivity of saturated frozen soils is much lower than that of the same soils unfrozen. As the temperature falls, the hydraulic conductivity decreases because of the freezing of capillary water and the decrease in thickness of the unfrozen water films. The hydraulic conductivity does not fall to zero, however. Some permeability to water remains, even at quite low temperatures.

The efficiency of ice segregation varies with soil texture. Silty soils generally account for most engineering problems; they are the most highly susceptible to ice segregation. Under comparable conditions, soils

finer than silts generally contain less segregated ice and are less susceptible to frost heaving, probably because their hydraulic conductivities are so much lower, both in the saturated and the unsaturated condition, than those of coarser soils. The hydraulic conductivity of a given freezing soil ranges between the low values observed during a freezing cycle and the high values observed during a thawing cycle. Hysteresis effects on all soil properties are amplified by freezing and thawing. Thus, the differences between wetting and drying cycles are amplified, as are disparities between laboratory samples. Variations in the hydraulic and thermal properties of soils, coupled with variations in available soil moisture, result in variations in the depth of frost penetration and the amount of frost heave. The differential rates and amounts of frost heave from one adjacent location to another often are troubling aspects of frost heaving. In fact, these differences are probably the principal cause of most geotechnical problems resulting from frost heaving.

Heaving always results in irreversible changes in the structure of soils. Mechanical properties are greatly affected when ice lenses develop and the redistribution of soil water occurs. Compressible soils such as clays consolidate beneath a growing ice lens if the effective stress increase caused by the pore water suction exceeds the past maximum effective stress. This may not cause a thaw weakening problem during subsequent thawing, if melt water can be readily drained away. In fact, it is sometimes possible to take advantage of this process to improve strength and consolidation characteristics. However, if the possibility of draining during thaw is not recognized, estimates of overconsolidation by freezing can be highly inaccurate (Chamberlain and Blouin 1978). This can be a problem in applications of artificial ground-freezing techniques, where a common, erroneous assumption is that an estimate of thaw consolidation can be taken as equal to measured frost heave.

Ice segregation also causes changes in the hydraulic conductivities of soils in another way. The structural changes caused by the increased effective stress within the "frozen fringe" or in the unfrozen zone beneath the freezing front, are known to cause large increases in the thawed hydraulic conductivities of soft clay soils, whereas the disintegration of soil aggregates that is caused by ice segregation in some silty soils results in lower hydraulic conductivities. Thus, ice segregation and frost heaving during repeated freeze-thaw cycles cause changes in soil structure that also affect hydraulic conductivity.

Most studies on frost heave have been concerned primarily with near-surface conditions and relatively short time periods. This is appropriate in the case of the seasonal frost heave of roads and airfields, and in the formation of needle ice in the vegetation mat and the soil surface. The potential of damage to a buried chilled gas pipeline by continual frost heave during the service life or to a buried liquefied gas tank is a more recent concern involving longer times and greater depths. This new perspective has served to broaden and extend considerations of frost heaving phenomena.

Massive Ground Ice

Excess ice produced by ice segregation is by no means confined to small ice lenses at shallow depths. There is abundant evidence from natural exposures along river banks and coastal bluffs, drilling, resistivity and gravity surveys (Rampton and Walcott 1974) to show that large tabular bodies of ice, commonly referred to as massive ice, are widespread in some permafrost areas of North America and the Soviet Union. Most of the massive ice so far encountered in drill holes lies within the 5- to 40-m-depth range (Mackay 1971, 1973ab), but some massive ice has been found at depths exceeding 100 m (Dubikow 1982). The area of some discrete bodies of massive ice may exceed 0.5 km^2.

The thickness of massive ice accumulations commonly ranges from less than 1 m to a maximum reported thickness of about 40 m. Massive ice bodies show every conceivable gradation in purity, from ice with no sediment at one extreme to high ice content sediments at the other extreme. In general, massive ice occurs near the basal portion of fine-grained material that typically is underlain by sand (Mackay 1973ab; Dubikow 1982). The fine-grained sediments in which the massive ice is found show a wide range of depositional environments such as basal till, lake clays, and marine clays. The massive ice is usually layered or stratified when viewed in natural exposures as if the ice had grown in situ and distended the sediments, in accordian-like fashion. Some of the massive ice is deformed. Folding can often be linked with differential loading.

There has been considerable debate as to whether the massive ice is buried glacier ice, segregated ice which has grown in place, or ice frozen from bulk water injected under pressure into frozen or unfrozen sediments and then frozen (e.g., Mackay 1971, 1973ab ; Karpov 1981; Koreisha et al. 1981; Dubikow 1982; Anisimova and Kritsuk 1983). Some massive ice is doubtless buried glacier ice. Indeed some exposures of debris-rich regelation ice known to have come from contemporary glaciers closely resemble massive ground ice. However, much of the massive ice probably is not buried glacier ice because (a) some massive ice is in unglaciated areas, (b) some massive ice is found in lacustrine and marine sediments or lies beneath topographic highs where burial would be difficult to explain, (c) the salinity of most of the massive ice is far greater than that of glacier ice, (d) the ice petrofabrics are similar to that of thin lenses of segregated ice, and (e) vertical profiles of the ionic content of the ice (water) from the ground surface to the sediments beneath the massive ice tend to show a common water origin for the ice (water), all of this is incompatible with a glacial origin.

Most massive ground ice probably is segregated ice frozen in an open system with the source water being under pressure (e.g., Karpov 1981). The common occurrence of massive ice in fine-grained sediments underlain by sands cannot be coincidental; it suggests rather that the sands provide

the open system for water under pressure needed to sustain segregated ice growth for a long period of time. Evidence for water under pressure can sometimes be seen in injection forms of pure ice such as dikes, sills, and irregularly shaped finger-like ice masses. The water quality analyses are in agreement with what would be expected from the growth of segregated ice by downward freezing (Anisimova and Kritsuk 1983; Dubikow 1982). The mineralization often increases with depth, presumably from solute rejection during freezing. Mineralization is also related to soil type and ice content. Oxygen isotope analyses have been used to obtain an indication of the water source of some of the massive ice (Mackay 1983ab). Data available for the Beaufort Sea coast indicate a cold freshwater source.

Mathematical Simulations and Numerical Modeling

As mentioned earlier, in ice segregation and frost heaving, transient processes of heat and mass transfer are coupled. Consequently, nearly all mathematical simulations contain a set of coupled partial differential equations describing the flow of heat and water. Although these equations are inherently nonlinear, during the course of the various mathematical treatments employed they typically become linear. The complexity of ice segregation and frost heaving requires numerical solutions, typically with finite differences applied in time, and either finite differences or finite elements applied in space. Hysteresis complicates the problem greatly. In fact, no heave simulations are yet known that are capable of dealing with hysteresis, although as already pointed out, hysteresis is known to be a common occurrence.

"Modeling" is a term that has become fashionable during the last two decades. "Mathematical modeling" refers to the development of mathematical representations based on physical concepts, instead of the development and construction of physical models. With the advent and present widespread availability of computers, mathematical modeling has become increasingly useful.

The objective of the modeler determines to a large extent the type of model developed. For example, engineering models tend to be primarily predictive; models developed for scientific purposes tend to be much more elaborate and to reflect an effort toward a complete understanding of key aspects of the natural processes involved. Scientists commonly take a constructionist approach and attempt to model every aspect of a process. Engineers often employ a systems approach that can include rather complete descriptions of certain physical and thermodynamic aspects of the system of processes, but in the absence of complete understanding they ultimately must resort to simpler, phenomenological descriptions of the processes. In some modeling efforts only the sign or order of magnitude change in an

output variable initiated by a parameterization change or alteration of an input variable or boundary condition is needed. This is a kind of "sensitivity testing" that is frequently employed when the system parameterization is weak due to inadequate information. In other cases detailed models are tailored for "site specific" design analysis.

It is possible for models to produce simulations that overreach understanding and invite self-delusion. Limitations in understanding the underlying processes and the great variety of circumstances under which frost heave occurs can lead to modeling excesses in addition to errors from other sources. It is also possible to construct models that are so elaborate that the parameters required cannot be adequately identified, specified, or measured. So many interacting processes may be included that cumulative errors of measurement and parameter specification, together with the practical limitations of testing and application, lead to grossly inaccurate results. Clear-cut conclusions may be impossible to extract from the results. Yet when the physical complexity is avoided by a series of oversimplifications, similar difficulties arise.

Notwithstanding these limitations, there is much promise in the modeling of ice segregation and frost heave. Future new models may become more complex in order to achieve closer fidelity to the processes and conditions involved. In any case, rapid progress is expected as the limitations and pitfalls are clearly recognized at each step and if applications are carefully chosen.

All frost heave models must handle the latent heat effects that accompany phase changes. These effects are extraordinarily large, and they can be troublesome. Crank (1981) provides a review of some practical methods for handling a simple phase change. If one assumes that the phase transition occurs as a step change in space, several methods recently developed to attack Stefan-type problems are useful, (Bonnerot and Jamet 1979; Crank and Prahle 1973; Lynch and O'Neill 1981). These methods faithfully model the phase discontinuity by ensuring that a numerical mesh boundary coincides with it. It is difficult to use approximate techniques and finite mesh spacings to deal with discontinuities, and even more difficult to deal with moving discontinuities. These techniques avoid approximation across the zone of highly concentrated activity and discontinuity and express latent heat effects through boundary or compatibility conditions.

Problems also may be approached by considering the occurrence of the phase transition as a step in macroscopic space but amenable to a smoothed numerical representation. However, some systematic artificial distention of the transition zone must be used such that energy is conserved. This approach is the essence of a variety of "enthalpy approaches" (Voller and Cross 1981). In finite element techniques latent heat effects may be spread over an entire finite element. Thus, the phase change locale is arbitrarily expanded to the model's scale of discrimination. This feature is similar in concept to some other related techniques such as the method of "excess degrees" of Dusinberre (1949) and the approaches of Doherty

(1970) and Hromadka et al. (1980, 1982). These approaches involve methods of repeatedly forcing computed freezing-zone temperatures back to a phase-change temperature as heat is withdrawn until the latent heat of that zone is exhausted. For situations in which the theory applied or the scale of analysis requires or involves a physically extended phase-change zone, a nonstep freezing-zone representation follows naturally (e.g., Taylor and Luthin 1978; O'Neill and Miller 1982; Miller 1981).

The principle of conservation of momentum (balance of stress) also must be incorporated in any frost heave model. Experimental evidence (e.g., Penner and Ueda 1979) shows that an increase in overburden pressure (accumulated weight on the freezing zone) diminishes the amount of heave produced by forces acting in the freezing zone, other things being equal. Overburden pressure is defined as a macroscopic net force per unit area that must be balanced by the combination of forces developed in each constituent of the freezing or thawing soil. If a weight is applied at the top of a soil column of unit area, the overburden pressure at the top equals this weight. At the bottom of the column the overburden pressure equals that weight plus the weight of the column.

The pressure in any phase or constituent usually differs from the overburden pressure. For example, if the pore water pressure in a saturated, coarse-grained, unfrozen soil is zero gage, then any weight on the soil and the soil column above must be supported entirely by stresses transmitted through intergranular contacts of the soil mineral. This is commonly known as the "effective stress." Microscopically, the stresses caused by intergranular contacts are quite complex. The details may be smoothed, however, and referred to simply as net intergranular force per unit area of soil. Usually, the overburden pressure is not fully balanced by the effective stress; pore contents participate as well. The macroscopic partitioning of pore stress, or so-called "neutral stress," into degrees of participation by the various pore constituents involves a microscopically complex interplay of stresses within the constituents, of energies at the interface, and of microscopic geometric configuration. Miller (1978) has suggested the use of a "stress partitioning factor" to weight the relative participations of macroscopic pore constituent stresses. Recent work (Snyder 1980) has provided some relevant experimental results, and this concept is an integral part of the model of O'Neill and Miller (1982, 1984).

Reasoning by analogy with unfrozen, swelling clays, Groenevelt and Kay (1977) have derived equations connecting overburden pressure, ice pressure, liquid water pressure, and the volume ratios of ice and liquid. Sheppard et al. (1978) refer to this approach in the presentation of their model but do not use it. Groenevelt and Kay (1980ab) have developed additional relationships between envelope, liquid, ice, and solid pressures from thermodynamic concepts.

One may question the validity of construing stresses within the system as having simple macroscopic pressure-like manifestations, especially when the stresses refer to an ensemble of grains or correspond

to forces within extremely thin adsorbed layers, tightly bound to the mineral particles (Takagi 1980; Miller 1980ab). It has been suggested that when water in the films pulls nearby water in while simultaneously pushing away the ice that forms, it exhibits a state of stress which is nonisotropic and otherwise complex (Miller 1980b; Vignes and Dijkema 1974; Vignes-Adler 1977). Derjaginn and Churaev (1978) also invoke surface layer forces to explain the expulsion of particles from adjoining ice. Romkens and Miller (1973) propose other descriptions of transfer processes in the film surrounding mineral particles embedded in ice. Osmotic pressure concepts also have been used (Miller 1980a), to explain film-water/pore-water/ice-pressure relationships. Much remains to be done before consensus is achieved. Consequently, defining, measuring, and utilizing the "right" pressures remain formidable obstacles.

The presence of unfrozen water in a heaving soil is taken into account in different ways by the various frost heave models now being developed. As indicated earlier, a major part of the unfrozen water flow takes place within the warmest, least frozen portion of the frozen soil. Ice segregation usually occurs within the frozen zone, slightly above a "frozen fringe." The frozen fringe is defined as the zone in which volumetric ice content increases from zero at the freezing front to nearly 100 percent at the location of the warmest lens (e.g., Hoekstra 1969ab; Loch and Kay 1978). The thickness of the frozen fringe is variable and depends on several factors. Two are particularly important: high overburden pressures and low temperature gradients both act to increase the thickness of the unfrozen fringe.

A well-known relationship that has been accepted as applicable to the ice-liquid mixture in frozen soil is the generalized Clapeyron equation

$$V_w \, d(P_w - \pi) - V_i \, dP_i = \frac{L}{T_o} \, dT \qquad (1)$$

where

L = latent heat of fusion of water (80 cal/g);

$V_i(V_w)$ = specific volume of ice (liquid water) (m^3/kg);

$P_i(P_w)$ = pressure acting on the ice (liquid water) (N/m^2);

T = temperature (degrees Kelvin);

T_o = freezing point of bulk water at atmospheric pressure (degrees Kelvin); and

π = osmotic pressure (N/m^2).

Recent derivations of this equation have been offered by Kay and Groenevelt (1974) and Loch and Kay (1978). This equation can be used to represent the equilibrium between pore ice and unfrozen water distributed

between two limiting conditions: at one extreme, water in thin films at the interface, with high overall pressure and high osmotic pressure; at the other extreme, unfrozen pore water with much lower overall pressure and osmotic pressure (Miller 1980ab). When applied macroscopically, P_w and π in this equation are regarded as pertaining primarily to pore water. The difference ($P_w - \pi$) is regarded as unique for the medium in a given state.

A number of investigators have noted the experimental evidence linking this concept with capillary theory. When liquid and another water phase coexist in a configuration with curved phase interfaces (as in soil pores), there is a pressure difference between the two phases due to effects at the interface, expressible in general as

$$P_j - P_w = \omega_{jw}F(\Theta) \qquad (2)$$

where j is either "i" for ice in a saturated frozen soil, or is "a" for air in an unsaturated unfrozen soil, and Θ denotes unfrozen water content. The factor ω_{jw} is the surface tension (N/m) associated with the interface in question. $F(\Theta)$ is a function that appropriately represents the mean curvature of the microscopic phase interfaces.

The earliest applications of this approach treat $F(\Theta)$ as a constant. Some of the simpler more recent models still do so. For example, one may begin with a given ice pressure, assumed equal to the overburden pressure, as in some cases of pure, segregated ice near the top of the soil. The curvature function $F(\Theta)$ then is assigned a value in keeping with a pore size of some "critical radius" characteristic of the soil. This procedure yields a maximum pressure difference and a value of P_w, via equation (2). Given additional assumptions about the temperature and pressure behavior of the rest of the system, one can determine whether heave occurs and attempt to estimate heaving rate or maximum heaving pressures. Experience suggests, however, that this approach may be too simple and that it cannot satisfactorily account for observed heaving behavior.

Koopmans and Miller (1966) introduced the hypothesis that the phase interface configuration at a given unfrozen water content, Θ, could be regarded as the same for an unsaturated unfrozen sample and a saturated frozen sample, for a given soil. That is, the same function $F(\Theta)$ should apply in both cases. If frozen and unfrozen cases are compared, and P_w is kept at zero in each, then from (2) we expect

$$P_i/P_a = \omega_{iw}/\omega_{aw} \qquad (3)$$

From measurements on frozen and unfrozen samples of the same granular soil, Koopmans and Miller found this ratio to be about 0.45, which is in reasonable agreement with estimates of the right-hand side of equation (3), based on other measurements (Hesstvedt 1964). For a clay, however, this ratio was found to be unity, indicating that in extremely fine-grained soils, noncapillary effects dominate.

For unfrozen, unsaturated soils, curves representing $F(\theta)$ are called "soil moisture characteristic" curves. Values differ somewhat depending on whether a drying or wetting sequence is used to obtain the data (hysteresis effect). Neglecting the hysteresis effect, if air pressure is atmospheric, a unique curve relating moisture content and pressure in the liquid phase is obtained. In an analogous manner, in the case of frozen saturated soils, one may construct "freezing characteristic" curves by relating θ to two other variables: for example, either to P_i and P_w as in (2), or to P_w and T by substituting (2) into (1) (assuming that π is either known or negligible). The phase composition curves illustrated in Figure 3, relating the unfrozen water content, W_u, and temperature, are special cases of freezing characteristic curves. That θ should depend on both pressure and temperature corresponds to common experience with bulk water. At a given temperature, pressure melting may be induced by an increase in applied pressure. Similarly, at a given pressure, one may induce freezing or thawing by changing the temperature. In a heaving soil, both pressure and temperature are variable, and, in a complete description, both of these variables must be related to θ, simulta-eously.

Misunderstanding of the freezing characteristic curves is widespread For example, liquid pressure (gage) in a freezing soil cannot be obtained directly from an (unfrozen) moisture characteristic curve. Although research indicates that $F(\theta)$ may be the same in both cases, the ice-water surface tension, ω_{iw} must be used in (2) in place of ω_{aw}. Moreover, the quantity $\omega_{iw}F(\theta)$ yields values for the difference between the ice and liquid pressures; absolute values of each are arbitrary. In some modeling approaches, it is assumed that the ice pressure is zero (gage). For example, when overburden pressure is negligible, it may be thought that P_w can be determined from (2) with $P_i = 0$. This may not be the case; if the unfrozen water is in tension, substantial compressive stress may still have been built up in the ice, in accordance with the capillary pressure jump between the phases and the two pressures may easily combine to produce very low neutral stress. In principle, an infinite number of combinations of liquid and ice pressure are possible for any value of θ.

The ability to specify pressures is extremely important in heave models because of the manner in which the flow of water must be handled. The flow equation is constructed upon the assumption of the Darcy law for fluid flow and the conservation of mass. For saturated unfrozen soil, the water content, θ, is equal to the porosity. Hence, it changes with time only if the soil matrix deforms. Given appropriate boundary and initial conditions together with the parameter values, the resulting flow equation can be solved.

For unsaturated soil, the water content, θ, is not equal to the porosity. Therefore, an appropriate relationship is required to relate the water content and the pressure in the water phase, P_a. One may either express θ in terms of $(P_w - P_a)$, where P_a is atmospheric pressure, or one may express P_w terms of θ and P_a. In the unfrozen

unsaturated case, P_a is essentially constant, and a unique relation between P_w and θ results. From this result one may construct a soil moisture diffusivity parameter. In several frost heave models, soil moisture diffusivity is used in the same manner as is customary for frozen soil. However, the analogy is not exact. Unlike P_a, the ice pressure is not uniform but varies from small values where the ice content is very low to higher values, which approach overburden pressures, at locations where ice segregation is occurring.

Expressions of soil-water flux-drive relationships that are more general than Darcy's law, have been developed by Kay and Groenevelt (1974) and Groenevelt and Kay (1974). The methods of irreversible thermodynamics are used to provide general expressions for mass fluxes in terms of both temperature and pressure gradients, each multiplied by phenomenological transport coefficients. Additional vigorous thermodynamic analyses are brought to bear to express relationships between the phases, the nature of the transport coefficients, and the coupling of transport processes. Such detail and rigor, however, are not features of existing frost heave models.

The process of regelation also is involved. It has been observed that applied pressures or temperature gradients may cause relative motion between ice and small particles completely enveloped in the ice (Gilpin 1979, 1980; Romkens and Miller 1973). The processes involved are not completely understood, but it is clear that ice must melt on one side of each enveloped particle and the resulting liquid be transported around the sides of the particle to a location where it refreezes (Hoekstra and Miller 1965; Hoekstra 1966; Anderson and Morgenstern 1973). It has been suggested that this process operates in heaving soils, so that net movement of ice, as well as movement of the unfrozen water, may occur (Miller 1978). That regelation occurs gives rise to a number of questions. For example, when a pressure gradient is imposed on a frozen soil sample to obtain hydraulic conductivity values for modeling purposes, to what extent does movement of the ice contribute to the observed mass transfer? What other factors might affect the ice movement rate that were absent or unknown during the hydraulic conductivity measurement but may be significant in other situations for the parameter values one applied? Horiguchi and Miller (1980) report difficulties in their efforts to distinguish between hydraulic flow and transport by regelation in permeameter experiments. Additional concepts or rules dealing with this process may be needed to characterize adequately the dynamics of pore ice movement and to relate it to the flow of unfrozen water and heat.

Mathematical expressions for heat flow needed in frost heaving simulations begin with the principle of energy conservation. Attempts continue to develop rigorous procedures for averaging accepted microscopic statements of the conservation laws applicable to a single phase (e.g., liquid within a pore) in such a way that valid macroscopic relations result (Gray and O'Neill 1976; Hassanizadeh and Gray 1979ab, 1980).

Frost heave models usually begin by assuming a basic heat balance relating the volumetric (sensible) heat capacities of all phases, the rate of change of sensible heat per unit volume, the divergence of the flux of sensible heat and the latent heat effects. Examination of enthalpy equations pertaining to simple, single-phase systems reveals that certain terms may or may not be significant (e.g., Bird, et al. 1960). Fourier's law of heat conduction usually is used to express the rate of heat flow. The expressions one uses depend in part on assumptions about the mass transfer processes.

The first efforts at modeling coupled heat and moisture transport in freezing soils were by Harlan (1973) and Guymon and Luthin (1974). They used continuum-deterministic approaches, sometimes referred to as conceptual or physics-based approaches which have been preferred by most researchers. Generally, mathematical models have included simultaneous heat and moisture transport in a one-dimensional column. Although there is almost total agreement that these two processes must be included in any model of frost heave, there is considerable uncertainty and disagreement on how to model ice segregation.

Guymon et al. (1981ab) have discussed a systems approach to the mathematical modeling of frost heave shown in Figure 4. The prototype system $\underline{S}$ (e.g., a laboratory soil column or field soil column) is subject to excitations (or inputs), x, that are spatially and temporally distributed. Spatially and temporally distributed outputs, y, then are obtained. Inputs or boundary conditions may be subfreezing temperatures, water-table location, and surface surcharge (overburden). Outputs may be frost heave, soil pore pressure, temperatures, or ice content. Because it usually is impossible to measure x exactly, subsystem $\underline{X}$ indicates a model process to determine an index, x', of x, which has some error. In most cases, x is lumped in space while preserving to the extent possible, any temporal low-frequency dynamic characteristics of x. Model $\underline{M}$ is assumed to be a deterministic model based on the continuum assumption. Certain parameters arise in the model derivation that purport to characterize $\underline{S}$ (e.g., thermal conductivity or hydraulic conductivity). Subsystem $\underline{P}$ indicates this modeling or sampling process, which yields imperfectly known parameters, p_i. Therefore, model outputs, y', will be imprecise but may be compared to imperfect observations of y for some bounded time period to determine model uncertainty, e(t), where

$$e(t) = y'(t) - y(t)$$

(One considers y as lumped in order to make this computation.) Modeling uncertainty is arbitrarily grouped into four general areas:

1. Errors a_1 due to the choice of $\underline{M}$, which includes the choice of numerical analog;

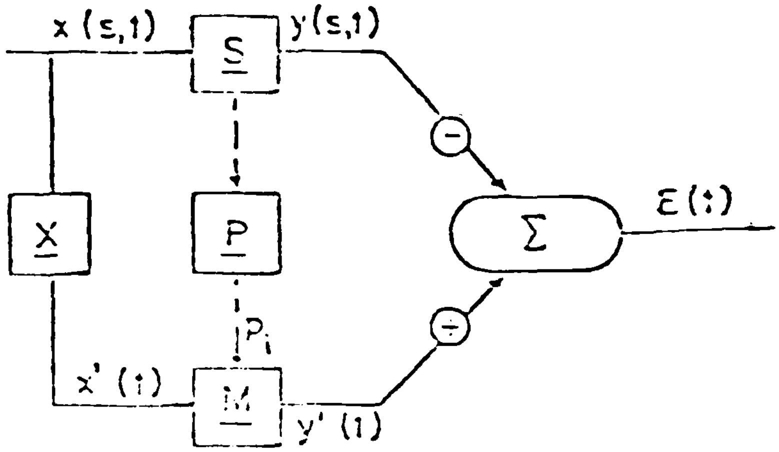

Figure 4. Schematic diagram of modeling concept.

2. Errors a_2 due to spatial and temporal discretization and averaging;

3. Errors a_3 due to boundary conditions (i.e., choice of $\underline{X}$) and due to choice of initial conditions;

4. Errors a_4 due to the selection of p_i (i.e. choice of $\underline{P}$).

The total model uncertainty is some function of the a_i errors

$$e(t) = e(a_1, a_2, a_3, a_4),$$

where the a_i errors may be interrelated and e may be nonstationary.

Conceptual mathematical models of nonlinear soil transport processes may be made linear to evaluate their reliability against closed-form analytical solutions. Basically, this process is a standard analysis technique to ensure that there are no computer code errors, as most models must be solved on a computer. The complexity and nonlinear nature of models that deal with most soil phenomena require comparison to prototype situations to evaluate model precision. Comparison of model simulations to prototype data also permits evaluations of parameter and boundary-condition sensitivity.

It might be argued that precise knowledge of ice segregation processes will eliminate uncertainty problems; however, this result is unlikely. Significant measurement errors for many of the parameters that are incorporated in mathematical models of frost heave (e.g., hydraulic conductivity of soil) are inevitable. Consequently, modeling or analysis of frost heave must proceed with a constant appreciation of the variability of key factors. Ideally future models will take that variability into account.

Current Models

The limitations of all current models may be characterized collectively by simply stating that at present no model enjoys universal or general acceptance. Frost heave modeling has been reviewed by Guymon et al. (1980), Hopke (1980), and more recently O'Neill (1983). Among the simplest heave models are those of Arakawa (1966), Outcalt (1980b), and Dudek and Holden (1979). Each of these presupposes simple heat conduction above and below a narrowly circumscribed freezing front. A heat balance is constructed at the front involving the divergence of the sensible heat flux, the upflow of unfrozen water, and the release of latent heat associated with any ice lens or advance of the freezing front. If more heat is withdrawn from the freezing front by conduction than can be accommodated by the upflow of water, then the freezing front in these models advances. Otherwise, heave occurs. In the model of Outcalt, the flow is driven by a pressure gradient depending always on a single suction value selected as representative of a partially frozen condition. A single, representative hydraulic conductivity value is also applied.

In Dudek and Holden's approach, a similarly simple liquid drive is constructed, in terms of the pressure drop across a small space near the freezing front. This maximum liquid pressure drop corresponds to a unique freezing point (temperature) depression value, dependent on the identity of the soil. Critical pore radius concepts are applied to determine whether possible heave will in fact result, given a heat deficit in the freezing zone. These models have the virtue of simplicity. They are limited, however, by not including much of the physics specific to the processes of ice segregation. Among other things, neither ice nor overburden pressure effects are accommodated.

Harlan (1973) and Guymon and Luthin (1974) develope models of one-dimensional heat and moisture flow in partially frozen soil based on simultaneous heat and moisture flow equations. Although somewhat different ancillary equations are used for subprocesses, the general approach is similar. Harlan has used finite differences, and Guymon and Luthin have used finite elements. Jame and Norum (1980) used Harlan's (1973) model to solve more extensively coupled heat and moisture flow equations, applied in greater detail over a portion of freezing soil in which coupled transfer and phase change take place. In an unsaturated situation, mass flow is driven by a gradient in unfrozen water content. This gradient is expressed in the same way in both the frozen and unfrozen zones, i.e., a diffusivity formulation is used. An experimental curve is employed for relating unfrozen moisture content to temperature. Thermal and mass transfer coefficients vary throughout the soil according to conditions, with soil moisture diffusivity values patterned after data from unfrozen unsaturated soil. Laboratory measurements of water redistribution in a freezing soil are estimated well if the calculated liquid flux is reduced by an impedance factor ranging between 1 and 1000.

Transport parameters are assumed to vary continuously in space, because no ice segregation occurs and the soil remains a continuous medium.

A number of other researchers have followed essentially similar lines, modifying ancillary relationships and adding various features including segregation and heave in open systems (Outcalt 1976, 1977; Taylor and Luthin 1978; Sheppard et al. 1978; Berg et al. 1980). Instead of soil moisture diffusivity, Outcalt has used an unsaturated hydraulic conductivity, with the same dependency on pressure in both frozen and unfrozen zones. Liquid pressure is obtained from an expression in terms of the vapor pressure of ice, and liquid content is taken as a function of pressure only. Simulations of hypothetical systems are qualitatively reasonable. Taylor and Luthin compare computations with the measurements of Jame and Norum (1980) and also those of Dirksen and Miller (1966). They too reduce a liquid diffusivity applied in the frozen soil by a large impedance factor. The calculated mass content in the system is continually readjusted through an "R coefficient," to preserve consistency. Sheppard, Kay, and Loch express phase change rate as the sum of liquid accumulation rate and the divergence of liquid flux. Temperature and pressure are related by the Clapeyron equation, with the ice pressure assumed uniformly zero.

Guymon et al. (in press) have continued the development of a model first described by Berg et al. (1980) and Guymon et al. (1980). This model solves one-dimensional coupled heat and moisture transport equations. Phase change effects are estimated by a simple heat balance bookkeeping procedure for a freezing or thawing zone. Consolidation effects are modeled on the basis of Terzaghi's theory. The model includes surcharge and overburden effects by modeling pore pressures as ice segregation is occurring. The model requires the calibration of hydraulic conductivity values by forcing model output to fit laboratory test results at certain times. The deterministic model is cascaded with a probabilistic model (Guymon et al. 1981b) to account for parameter uncertainty. Verification of results against laboratory and field data are reported by Guymon et al. (1981a, and in press) and Hromadka et al. (1982). This one-dimensional model has also been extended to simple two-dimensional cases (Guymon and Hromadka 1982).

Within relatively homogeneous laboratory soil columns, lenses sometimes are often observed spanning much or all of the column width. In such cases, these lenses represent substantial discontinuities in the soil and must intercept most or all of the incoming liquid flow. Such radical changes in the scale of phenomena to be simulated challenge traditional soil modeling methods. In one-dimensional modeling a lens blocks flow, but in the three-dimensional situation lenses are discontinuous. Thus, as in the modeling of atmospheric turbulence, for example, only three-dimensional models are completely realistic. Attempts to parameterize three-dimensional processes in one- or two-dimensional model space are always implicit in model building, but the implications usually are not considered explicitly. This fact, together with previously mentioned

parameterization problems, appears to set an upper boundary on model validity.

The most physically elaborate numerical models are those of Hopke (1980), Gilpin (1980), and O'Neill and Miller (1982, 1984). Hopke's model relates temperatures and pressures using the Clapeyron equation and capillary concepts. Use of the same freezing characteristic function in the unsaturated frozen and unfrozen zone, together with assumptions about the ice pressure behavior, leads evidently to discontinuities that complicate the computations. Ice pressure is assumed to be zero at the freezing front and also within the frozen soil when pores are not completely filled with liquid water and ice. Ice pressure builds as freezing progresses, and heave is allowed to occur wherever ice pressure equals the overburden pressure. Overburden is included in such a way that simulated results agree qualitatively with observation, in that increased overburden reduces heave. Unlike previous models, Hopke's can predict expulsion of water from the freezing zone, even while heave is occurring. This phenomenon has actually been observed particularly in early time in a freezing sequence in which phase change is most rapid. High overburden pressures were observed to amplify this effect. Usually the liquid flow reverses later in the sequence when freezing slows, ultimately flowing into the freezing zone. Transport is simulated through a frozen fringe that does not heave. Predictions based on estimated soil parameters are compared with data from Penner and Ueda (1977ab), and are qualitatively good, although calculated heave is overestimated for conditions that entail a thin fringe. Hopke speculates that other mechanisms not accounted for in the model must limit heave under this circumstance.

An attempt to model geological time scales has recently been carried out by Outcalt (1982). This model generated massive ground ice near the base of the active layer and clearly illustrated the effects of water migration and heave in permafrost underlying the active layer.

RESEARCH NEEDS

As outlined in the preceding section, the widespread availability of computers has stimulated mathematical modeling of ice segregation. It is apparent that this effort has fully exploited and now has overreached the available field and laboratory data base. Much more effort should be expended in obtaining good laboratory and field data to sustain the continued development and verification of mathematical models. More detailed field studies, including studies of the stratigraphy of massive ground ice phenomena and long-term disturbance experiments, can furnish valuable information.

There is widespread agreement that the models of ice segregation need to include better established equations of heat and moisture flow as well as realistic heave and pressure submodels. Beyond this general agreement, however, there appear to be strongly divergent views on a number of fundamental questions that are the essence of the mathematical modeling of ice segregation. For example, the form and structure required of models are uncertain. There is no agreement on what must be included or can be omitted and how perceived subprocesses are computationally linked. Neither is there agreement on the mathematical form required for certain processes. For example, some investigators insist on a "complete" thermodynamic expression for freezing characteristics; others are satisfied with a constant value with respect to temperature to represent unfrozen water content. In addition, there is disagreement on how to account for surcharge and overburden pressures in freezing zones where ice segregation is occurring.

Conceptual as well as procedural difficulties obstructing further progress are numerous and formidable. It is not certain that a deterministic formulation adequate for either the scientific or engineering needs can be developed. Progress has been achieved, and exhaustive testing and verification of the most promising formulations to establish the degree of applicability to engineering analysis and design is an obvious current need. This activity will require both large-scale field tests and extensive laboratory tests coordinated to as high a degree as possible.

Although, as emphasized in the Introduction, there is general understanding and agreement on most of the qualitative aspects of the processes involved in ice segregation and frost heaving, intensive efforts carried on at many research centers throughout the world concerned with

constructing numerical simulations of ice segregation and frost heaving have culminated in a clear identification of a number of areas of controversy and difficulty. There follows a summary listing of topics of particular significance and concern, some expressed as questions with abbreviated comments.

- How can the overburden pressure be related to the state of water (liquid, ice, vapor) and the soil matrix?

  - The overburden pressure is regarded as being uniformly transmitted to the ice at least within ice lenses.
  - The overburden pressure is regarded as being added to the liquid pressure without partition.
  - The overburden pressure is regarded as being carried entirely by matrix without partition among the ice or liquid phases.
  - The overburden pressure is partitioned between ice, liquid, and matrix. If so, then there is a need for (a) direct measurements of partitioning coefficients and (b) scaling (indirect measurements of partitioning coefficients).

- Do relationships between overburden pressure and other parameters (e.g., liquid pressure, ice pressure) under static conditions also apply under dynamic (flux) conditions?
- What are the most appropriate measurable parameters for the characterization of frost heaving?
- For purposes of numerical simulation, what are the criteria for ice lens formation?

  - Volumetric total water content exceeds original porosity.
  - Volumetric water content equals or exceeds porosity, and effective stress goes to zero.
  - Freezing process reaches state where it is dominated by pores of a critical size.

- What is the effect of solute rejection during the freezing process on ice segregation?
- How should coupled transport processes (liquid water, ice, vapor, heat) be expressed in simulations of frozen soils with and without ice lenses?

  - Deterministic approach: employ average values for transport coefficients and driving forces.
  - Stochastic approach: employ scaling, frequency distributions, autocorrelation theory, etc.

- Continuing debate and differing opinions characterize the attempt to classify frozen soils and their properties and internal states.

Geotechnical engineers apply terminology and parameters suitable to unfrozen soils and transfer them to frozen soils; others consider this approach inappropriate and use different procedures.

- Uncertainty exists regarding processes in continuous long-term frost heave versus seasonal frost heave, as they relate to such things as the long-term stability of cold, buried structures. In certain cases, continuous long-term frost heave is possible and will seriously threaten certain structures. Resolution of this issue requires (a) improvements in the theory of heave processes and (b) long-term field and laboratory studies.
- How can the location of lens formation within the frozen zone be accomplished, and the processes associated with "secondary heaving" be better defined?
- What are the maximum theoretical frost heaving pressures and the maximum observable frost heaving pressures, and how are these pressures related (a) to displacement and to time, (b) to the concept of "shut-off pressures", and (c) to maximum observable frost heaving pressures in laboratory and field settings?
- To what extent does water and ice migration in frozen ground occur, and how can this movement be measured reliably?
- During frost heaving, what are the sources of the energy involved, and how are these energies partitioned?
- How is regelation involved in the buildup and maintenance or sustenance of frost heaving pressures? As ice moves in the soil, does it move as a rigid block, passing over soil grains by regelation?
- What determines and governs the physical processes of ice nucleation in freezing soils?
- Do soil materials have characteristic frequency distributions and autocorrelative functions that are useful in describing their characteristics and variability?
- In short-term laboratory simulations of long-term ice segregation and frost heaving, what is the best technique for dealing with or scaling time?
- How is the added complexity of solute redistribution dealt with? More information on solute rejection and partitioning is needed.
- Can we adequately determine pore water suction during freezing and make determinations of soil consolidation with models?

Research designed to answer these crucial questions and thus add to our current understanding is needed to allow further progress in numerical simulations and the mathematical modeling of ice segregation and frost heaving and to enlarge our current understanding of the basic governing principles and processes.

CONCLUSIONS AND RECOMMENDATIONS

As a result of this study, a number of recommendations intended to provide guidance and assistance to federal and state agencies, private firms, and consortia that are active in funding or conducting research in the scientific and engineering aspects of ice segregation and frost heaving can be advanced. The information contained in this report, together with these recommendations, can help accelerate progress toward a more fundamental understanding of the phenomena associated with ice segregation and frost heaving and can help to facilitate application of an improved understanding to the planning, design, and maintenance programs associated with buildings, facilities, airfields, roads, and utilities, wherever frost heaving is a hazard.

The processes of ice segregation and frost heaving are complex. Certain portions of the problems involving coupled and interrelated processes present unusually formidable difficulties; it is on these that future effort should concentrate.

1. We recommend a continued concentrated research effort to understand the basic processes of ice segregation and frost heaving. Among the most critical unresolved questions and problems are:

- What are the basic relationships that govern the coupled transport processes (liquid water, heat, water vapor, ice) in ice segregation?
- How are overburden pressures related to ice segregation and frost heaving?
- Do the same basic relationships between overburden pressure and other parameters that characterize frost heaving apply under both static and dynamic conditions?
- What are the important basic principles and relationships governing long-term, as distinct from seasonal, frost heaving?
- What are the maximum attainable frost heaving pressures under long-term frost heaving conditions and how are they best controlled?
- What are the sources of the energy involved in frost heaving and how are they partitioned among the coupled processes involved?
- Solute redistribution occurs during ice segregation. More information is needed on the basic principles and proccesses.
- How is regelation involved in the buildup and maintenance of frost heaving pressures?

• What determines the temperatures and governs the phenomena of ice nucleation in freezing soils?

2. We recommend a continued intensive research effort to develop predictive numerical models that by simulating ice segregation and frost heaving can provide a means of consolidating all the processes and interactions involved. This will continue to aid in attaining a better and more complete understanding of frost heaving and for assessing its impact in various settings and circumstances. Among the most critical unresolved questions and problems pertaining to this aspect are:

• What are the essential and most appropriate parameters required?
• What are the essential criteria for ice segregation and the formation of ice lenses or enlarging ice masses?
• How should the coupled transport processes (heat, liquid water, water vapor, ice) be most effectively and appropriately formulated?
• How can the locations of enlarging ice lenses be most effectively specified and the processes of "secondary heaving" be better defined?
• Must the partition of energy among the various processes involved be specifically recognized and dealt with? If so, how?
• Are there useful characteristic frequency distributions and auto-correlative functions that can be generally agreed upon and worked into the various approaches to numerical simulations of frost heaving?
• In both seasonal and long-term simulations, what is the best technique for dealing with and scaling time?

Progress in solving all of these difficult and controversial questions is crucial to the attainment of a complete, rational theory of ice segregation and frost heaving. Such a theory, however, is needed for the improved mathematical simulations and numerical models capable of producing reliable assessments of frost heaving hazards in a variety of locations and circumstances where engineering design and construction must take place.

3. A common terminology, clearly relating to and bridging existing terminologies in the various disciplines, is needed to address the phenomena of ice segregation and frost heaving and to facilitate a more efficient and rational discussion of the processes, consequences, and methods of ameliorating their adverse effects. Therefore, we recommend that the several national scientific and professional societies involved give explicit attention to this need and encourage efforts to achieve an appropriate, common terminology.

4. Improved instrumentation to measure the critical phenomena associated with ice segregation and frost heaving is needed. Most of this instrumentation must be developed for unique uses and circumstances. We recommend that this need be explicitly recognized by funding bodies and that the design and fabrication of needed instrumentation, in addition to

the procurement of more conventional and readily available instrumentation, be a prominent part of the budgets of ice segregation and frost heaving research projects.

5. We recommend that field and laboratory studies of ice segregation and frost heaving be better coordinated in order to accelerate progress in the verification and improvement of the various mathematical models currently available and to accelerate the quantification of the functions and parameters of the various interrelated processes. We recommend that this coordination be a part of the protocol of all major research projects dealing with problems of ice segregation and frost heaving.

In recent years professional societies have organized several conferences to facilitate the exchange of knowledge and data on the problems of ice segregation and frost heaving. However, much more could be done at relatively little cost during the planning stages of major research projects. The needed coordination could be accomplished without compromising the essential requirements of obtaining and managing proprietary data. Indeed, if attention is given to this recommendation, the value of the proprietary information obtained could be vastly increased.

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